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# SO(10) Unified Theories in Six Dimensions

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## Abstract

We construct supersymmetric models of  $SO(10)$  unification in which the gauge symmetry is broken by orbifold compactification. We find that using boundary conditions to break the gauge symmetry down to  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$  without leaving unwanted massless states requires at least two extra dimensions, motivating us to work with 6D orbifolds.  $SO(10)$  is broken by two operations, each of which induces gauge-breaking to either the Georgi-Glashow, Pati-Salam, or flipped  $SU(5) \otimes U(1)$  subgroups; assigning different unbroken subgroups to the two operations leaves only the standard model gauge group and  $U(1)_X$  unbroken. The models we build employ extra-dimensional mechanisms for naturally realizing doublet-triplet splitting, suppressing proton decay, and avoiding unwanted grand-unified fermion mass relations. We find some tension between being free of anomalies of the 6D bulk, accommodating a simple mechanism for generating right-handed neutrino masses, and preserving the precise prediction of the weak mixing angle.

# 1 Introduction

The successful prediction of the weak mixing angle in the minimal supersymmetric standard model (MSSM) is a compelling hint for new physics. The most direct interpretation of this hint is that of low energy supersymmetry and an energy desert, with no additional physics, extending between the TeV scale and the unification scale  $M_U \sim 10^{16}$  GeV [1].

How can nature be described above  $M_U$ ? One possibility is that it is described by a grand unified theory (GUT) [2, 3]. Grand unification offers an elegant explanation of the quantum numbers of the standard model quarks and leptons, but raises other new questions. These include the details of the gauge symmetry breaking, the origin of doublet-triplet splitting, and the reason for non-observation of proton decay.

There has been much recent interest in addressing these issues in the context of grand unified theories with extra spacetime dimensions [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. These recent models apply methods that first appeared in string-motivated work [16]: the gauge symmetry is broken by identifications imposed on the gauge fields under the spacetime symmetries of an orbifold, and doublet-triplet splitting occurs because the orbifold compactification projects out the zero modes of the colored components of the Higgs multiplets. The absence of proton decay induced by dimension five operators can also be given an intrinsically extra dimensional explanation involving the form of the mass matrix for the Higgsino Kaluza-Klein modes [6].

These ideas have been used to build complete and realistic 5D models of supersymmetric  $SU(5)$  unification on an  $S^1/Z_2$  orbifold [4, 5, 6, 7, 8]. The purpose of this paper is to explore whether similar ideas can be used to build simple model based on the  $SO(10)$  gauge symmetry. One motivation for considering  $SO(10)$  carries over from the 4D case:  $SO(10)$  allows an entire generation of quarks and leptons to be unified in an irreducible spinor representation. This representation includes a right-handed neutrino, so that  $SO(10)$  also provides a natural framework within which the see-saw mechanism [17] can be realized. In the context of extra dimensional models, we will find that working with  $SO(10)$  also illustrates how interesting group-theoretic structure can arise on orbifolds. For instance, the identifications we impose on the gauge fields under spacetime symmetries to break  $SO(10) \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$  naturally lead to fixed points in which only the Pati-Salam  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$  [18], Georgi-Glashow  $SU(5) \otimes U(1)_X$  [2], or flipped  $SU(5)' \otimes U(1)'_X$  subgroups [19] of  $SO(10)$  are preserved. What is the minimum number of extra dimensions required to break the gauge symmetry through orbifold compactification? In the  $SU(5)$  case a single extra dimension is sufficient. We will find that the larger  $SO(10)$  gauge group requires at least two extra dimensions for the orbifold compactification to break the unified symmetry without leaving extra massless states coming from the higher-dimensional supersymmetric vector multiplet. Thus the models we construct will be six dimensional.

The outline of the paper is as follows. In section 2, we consider the group theoretic structure

of  $SO(10)$  gauge symmetry breaking on a torus. The basic ideas discussed in that section are then applied in the rest of the paper to construct three different orbifold models. The first, presented in section 3, is a theory with  $N = 1$  supersymmetry on a  $T^2/Z_2$  orbifold. We will find that this orbifold provides a natural setting for doublet-triplet splitting and for extra-dimensional mechanisms for relaxing unwanted grand unified fermion mass relations. It also accommodates simple ways of breaking and communicating the unbroken  $U(1)_X$  gauge symmetry left after orbifolding to give right-handed neutrino masses. Unfortunately the theory is anomalous: irreducible gauge anomalies of the 6D bulk are easily canceled, but mixed gauge-gravitational anomalies remain. This motivates us to construct anomaly-free theories with 6D  $N = 2$  supersymmetry in sections 4 and 5. The model of section 4 is constructed on  $T^2/Z_6$ , with only the 6D  $N = 2$  vector multiplet allowed in the bulk. In this model there are no colored Higgs multiplets: matter and Higgs are localized to a fixed point that preserves only the Pati-Salam subgroup of  $SO(10)$ , and the Higgs doublets are contained in the  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  representation. The breaking of  $U(1)_X$  is straightforward but communicating it to standard model fields is not, and we will find that it is difficult to obtain right-handed neutrino masses (and to avoid  $SO(10)$  mass relations) in this model without facing a vacuum alignment problem. In section 5 we attempt to improve this situation by working with a  $T^2/(Z_2 \times Z'_2)$  orbifold, which has 5D “fixed lines” on which matter may propagate, without introducing 6D anomalies. The existence of these lines makes communicating  $U(1)_X$  breaking and correcting fermion mass relations much easier. However, this gain is likely at the expense of the precise prediction of the weak mixing angle. This issue is discussed in section 6. Our conclusions appear in section 7.

During preparation of this manuscript, we received Ref. [20], which also considers  $SO(10)$  breaking by orbifold compactification in six dimensions.

## 2 $SO(10)$ Gauge Symmetry Breaking on a Torus

In this section we consider the  $SO(10)$  breaking by orbifold compactifications. We begin by considering the case of a single extra dimension. The most general spacetime symmetries that can be used to compactify a single extra dimension may be taken to be a reflection  $\mathcal{Z}$  and a translation  $\mathcal{T}$  [11]. Fields propagating in the extra dimension may transform nontrivially under  $\mathcal{Z}$  and/or  $\mathcal{T}$ , as long as the bulk action is invariant under these operations and the transformations under  $\mathcal{Z}$  and  $\mathcal{T}$  are consistent:  $\mathcal{T}\mathcal{Z}$  and  $\mathcal{Z}\mathcal{T}^{-1}$  must act on fields in the same way because they induce the same motion in spacetime.

We first ask whether we can build a 5D  $N = 1$  supersymmetric model, in which  $SO(10)$  is broken by these transformations to  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$  (3-2-1-1). Such breaking requires both  $\mathcal{Z}$  and  $\mathcal{T}$  to have non-trivial gauge properties; for example,  $\mathcal{Z}$  and  $\mathcal{T}$  may be chosen to preserve  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$  (4-2-2) [18] and  $SU(5) \otimes U(1)_X$  (5-1) [2] subgroups of

$SO(10)$ , respectively. However, this results in the chiral adjoint of the 5D vector multiplet containing extra massless fields other than the states in the MSSM, so that the gauge coupling unification is spoiled.

In 6D this problem is immediately avoided, as there are now two translations,  $\mathcal{T}_1$  and  $\mathcal{T}_2$ , and they can be used to break  $SO(10)$  to  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ . The spacetime orbifold depends not only on the torus defined by  $\mathcal{T}_1$  and  $\mathcal{T}_2$ , but also on the non-freely acting symmetries used to identify parts of the torus. Here we assume that these non-freely acting orbifold symmetries preserve  $SO(10)$ , and hence for the purpose of describing the gauge symmetry breaking in this section we need not discuss them. In the next three sections different orbifolds are constructed, and in each case the orbifolding symmetries are used to ensure that there are no unwanted zero-mode states from the 6D vector supermultiplet. Actually, in the model of section 4, the orbifold symmetry breaks  $SO(10)$  while accomplishing this task, but the discussion here in terms of torus translations will be sufficient for illustrating the basic ideas we use for breaking  $SO(10)$  down to  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ .

The generators  $T^a$  of  $SO(10)$  are imaginary and antisymmetric  $10 \times 10$  matrices. We will find it convenient to write these generators as tensor products of  $2 \times 2$  and  $5 \times 5$  matrices, giving  $\sigma_0 \otimes A_5$ ,  $\sigma_1 \otimes A_5$ ,  $\sigma_2 \otimes S_5$  and  $\sigma_3 \otimes A_5$  as a complete set. Here  $\sigma_0$  is the  $2 \times 2$  unit matrix and  $\sigma_{1,2,3}$  are the Pauli spin matrices;  $S_5$  and  $A_5$  are  $5 \times 5$  matrices that are real and symmetric, and imaginary and antisymmetric, respectively. The  $\sigma_0 \otimes A_5$  and  $\sigma_2 \otimes S_5$  generators form an  $SU(5) \otimes U(1)_X$  subgroup of  $SO(10)$ , with  $U(1)_X$  given by  $\sigma_2 \otimes I_5$ . We choose our basis so that the standard model gauge group is contained in this  $SU(5)$ , with  $SU(3)_C$  contained in  $\sigma_0 \otimes A_3$  and  $\sigma_2 \otimes S_3$  and  $SU(2)_L$  contained in  $\sigma_0 \otimes A_2$  and  $\sigma_2 \otimes S_2$ , where  $A_3$  and  $S_3$  have indices 1,2,3 and  $A_2$  and  $S_2$  have indices 4,5. The generators of this  $SU(5) \otimes U(1)_X$  subgroup can be grouped as

$$SU(5) \otimes U(1)_X : \begin{array}{ccc} \sigma_0 \otimes A_3 & \sigma_0 \otimes A_2 & \sigma_0 \otimes A_X \\ \sigma_2 \otimes S_3 & \sigma_2 \otimes S_2 & \sigma_2 \otimes S_X. \end{array} \quad (1)$$

Here  $A_X$  and  $S_X$  denote the off diagonal pieces left over from  $A_5$  and  $S_5$ . A different  $SU(5) \otimes U(1)$  subgroup is formed by replacing  $\sigma_0 \otimes A_X$  and  $\sigma_2 \otimes S_X$  with  $\sigma_1 \otimes A_X$  and  $\sigma_3 \otimes A_X$ :

$$SU(5)' \otimes U(1)'_X : \begin{array}{ccc} \sigma_0 \otimes A_3 & \sigma_0 \otimes A_2 & \sigma_1 \otimes A_X \\ \sigma_2 \otimes S_3 & \sigma_2 \otimes S_2 & \sigma_3 \otimes A_X. \end{array} \quad (2)$$

This  $SU(5)'$  is known in the literature as flipped  $SU(5)$  [19]. It contains  $SU(3)_C$  and  $SU(2)_L$  but not  $U(1)_Y$ . Finally, it will be useful to list the generators that form the  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$  subgroup of  $SO(10)$ :

$$SU(4)_C \otimes SU(2)_L \otimes SU(2)_R : \begin{array}{cc} (\sigma_0, \sigma_1, \sigma_3) \otimes A_3 & (\sigma_0, \sigma_1, \sigma_3) \otimes A_2 \\ \sigma_2 \otimes S_3 & \sigma_2 \otimes S_2. \end{array} \quad (3)$$

The torus  $T^2$  has translation symmetries defined by two vectors  $e_1$  and  $e_2$  in the complex plane  $z = x_5 + ix_6$ . The translation symmetries of the torus identify two points in the complex plane,  $z_1$  and  $z_2$ , if  $z_1 = z_2 + me_1 + ne_2$  for integers  $m$  and  $n$ . Under the translation  $z \rightarrow z + e_i$ , the identifications imposed on the vector supermultiplet, which contains the gauge fields, are

$$V(z + e_i) = T_i V(z) T_i^{-1}. \quad (4)$$

In this paper we employ three possible forms for the  $T_i$  matrices. They are

$$T_{51} \equiv \sigma_2 \otimes I_5, \quad (5)$$

$$T_{5'1'} \equiv \sigma_2 \otimes \text{diag}(1, 1, 1, -1, -1), \quad (6)$$

$$T_{422} \equiv \sigma_0 \otimes \text{diag}(1, 1, 1, -1, -1). \quad (7)$$

Consider, for instance, the case  $(T_1, T_2) = (T_{51}, T_{5'1'})$ . With this choice for  $T_1$ , we have

$$\begin{aligned} T_1(\sigma_0 \otimes A_5)T_1^{-1} &= \sigma_0 \otimes A_5, & T_1(\sigma_1 \otimes A_5)T_1^{-1} &= -\sigma_1 \otimes A_5, \\ T_1(\sigma_2 \otimes S_5)T_1^{-1} &= \sigma_2 \otimes S_5, & T_1(\sigma_3 \otimes A_5)T_1^{-1} &= -\sigma_3 \otimes A_5. \end{aligned} \quad (8)$$

Thus, only  $SU(5) \otimes U(1)_X$  gauge fields are potentially massless once this transformation under the  $e_1$  translation has been imposed. On the other hand, the generators that commute with  $T_2$  are

$$\begin{aligned} &\sigma_0 \otimes A_3 \quad \sigma_0 \otimes A_2 \quad \sigma_1 \otimes A_X \\ &\sigma_2 \otimes S_3 \quad \sigma_2 \otimes S_2 \quad \sigma_3 \otimes A_X, \end{aligned} \quad (9)$$

while all the other generators anticommute with  $T_2$ . Comparing with Eq. (2), we see that the gauge fields with even parity under the  $e_2$  translation belong to  $SU(5)' \otimes U(1)'_X$  (5'-1'). Combining with the result from the  $e_1$  translation, we find that the only generators that are invariant under both translations are those of  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ . Therefore, only gauge fields from this subgroup will have massless zero modes, as desired. It is easily checked that taking  $(T_1, T_2) = (T_{51}, T_{422})$  or  $(T_1, T_2) = (T_{5'1'}, T_{422})$  leads to the same unbroken gauge group.

We finally summarize the group theoretic structure of  $SO(10)$ . The 45 generators of  $SO(10)$  can conveniently be assembled into seven groups, as shown in Fig. 1. The generators and their  $(T_{51}, T_{5'1'})$  parities are given by

$U(3)_C$	$\sigma_2 \otimes S_3; \sigma_0 \otimes A_3$	(+, +)
$SU(2)_L$	$\sigma_2 \otimes \sigma_{1,3}; \sigma_0 \otimes \sigma_2$	(+, +)
$T_{3R}$	$\sigma_2 \otimes \sigma_0$	(+, +)
$T_R^\pm$	$\sigma_{1,3} \otimes \sigma_2$	(-, -)
$SU(4)_C/U(3)_C$	$\sigma_{1,3} \otimes A_3$	(-, -)
$SU(5)/(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y)$	$\sigma_2 \otimes S_X; \sigma_0 \otimes A_X$	(+, -)
$SU(5)'/(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y)$	$\sigma_{1,3} \otimes A_X$	(-, +).

(10)

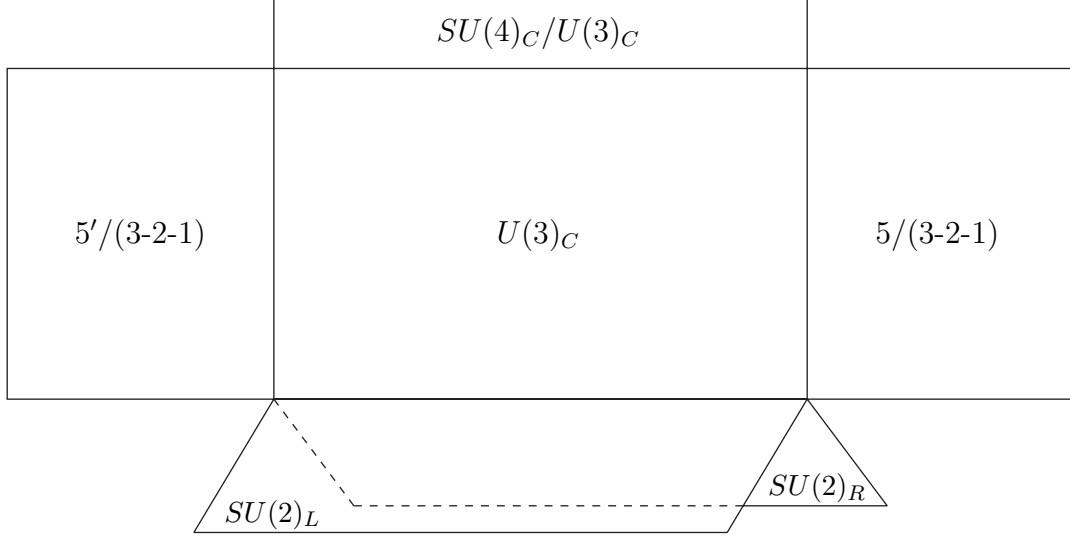


Figure 1: A convenient grouping of the  $SO(10)$  generators. The parities of the corresponding gauge bosons under torus translations are given in Eq. (10).

Here  $U(3)_C$  contains  $SU(3)_C$ , and  $SU(4)_C$  is the Pati-Salam group. The generators of  $U(1)_Y$  and  $U(1)_X$  are linear combinations of  $T_{3R}$  and  $U(3)_C/SU(3)_C$ .

This figure represents well the symmetries among the  $SO(10)$  generators. There are symmetries which interchange  $SU(2)_L$  and  $SU(2)_R$ , and  $SU(5)$  and  $SU(5)'$ . It is also useful in identifying the unbroken generators in patterns of  $SO(10)$  breakings; 4-2-2 type breaking corresponds to breaking generators in both left and right wings, and 5-1 ( $5'-1'$ ) type breaking to taking the body, the right (left) wing, front leg, and  $T_{3R}$  for unbroken generators. Therefore, it is easily seen that the combination of any two of 4-2-2, 5-1, and  $5'-1'$  type breakings leads to  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$  as the unbroken subgroup.

### 3 A Model on $T^2/Z_2$

The first model we consider is a 6D  $N = 1$  supersymmetric model with the extra dimensions compactified on a  $T^2/Z_2$  orbifold. In addition to the identifications under torus translations, two points  $z_1$  and  $z_2$  are identified if they are mapped into each other under a  $\pi$  rotation in the  $x_5$ - $x_6$  plane, *i.e.* if  $z_1 = -z_2$ .

For simplicity we take a rectangular lattice for the torus, so that  $e_1 = 2\pi R_5$  and  $e_2 = 2\pi i R_6$ . Consider the rectangle whose corners'  $z$  coordinates are 0,  $\pi i R_6$ ,  $2\pi R_5$ , and  $2\pi R_5 + \pi i R_6$ . The physical space may be taken to be the two-sided rectangle obtained by folding this rectangle in

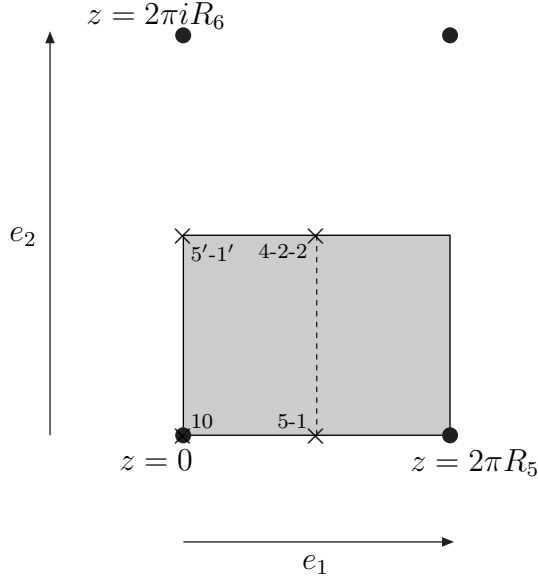


Figure 2: The  $T^2/Z_2$  orbifold in the  $z = x_5 + ix_6$  plane. Each orbifold fixed point is denoted by a cross and labelled by the non-trivial gauge transformations acting on it: 10 for  $SO(10)$ , 5-1 for  $SU(5) \otimes U(1)_X$ , 5'-1' for  $SU(5)' \otimes U(1)'_X$  and 4-2-2 for  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ . The physical space may be taken as the two-sided rectangle formed by folding the shaded region along the dotted line and then gluing together the touching edges.

half along the  $x_5 = \pi R_5$  line and then gluing together the edges that are touching one another (see Fig. 2). The orbifold fixed points are those that, under the  $\pi$  rotation, are mapped into points with which they were already identified under the translation symmetries of the torus. There are four orbifold fixed points on the space, whose  $z$  coordinates are 0,  $\pi R_5$ ,  $\pi i R_6$ , and  $\pi(R_5 + i R_6)$ . This theory could equally well be constructed with non-orthogonal vectors  $e_1$  and  $e_2$ , but the KK mode expansions are simplest for the orthogonal case.

### 3.1 Gauge fields in the bulk

Consider an  $SO(10)$  gauge multiplet propagating in this space. The fields may be described by a vector and chiral adjoint multiplet of 4D  $N = 1$  supersymmetry,  $(V, \Phi)$ . The bulk action is given by [21]

$$S = \int d^6x \left\{ \frac{1}{4kg^2} \text{Tr} \left[ \int d^2\theta \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.} \right] \right\}$$

$$+ \int d^4\theta \frac{1}{kg^2} \text{Tr} \left[ (\sqrt{2}\partial^\dagger + \Phi^\dagger) e^{-V} (-\sqrt{2}\partial + \Phi) e^V + \partial^\dagger e^{-V} \partial e^V \right] \Big\}, \quad (11)$$

where  $V = V^a T^a$ ,  $\Phi = \Phi^a T^a$ ,  $\text{Tr}[T^a T^b] = k\delta^{ab}$  and  $\partial = \partial_5 - i\partial_6$ .

Under the torus translations we identify

$$V(z + e_i) = T_i V(z) T_i^{-1}, \quad (12)$$

$$\Phi(z + e_i) = T_i \Phi(z) T_i^{-1}, \quad (13)$$

with  $(T_1, T_2) = (T_{51}, T_{5'1'})$ . Under the orbifold  $Z_2$  ( $\pi$  rotation), we identify

$$V(-z) = Z V(z) Z^{-1}, \quad (14)$$

$$\Phi(-z) = -Z \Phi(z) Z^{-1}, \quad (15)$$

with  $Z = \sigma_0 \otimes I_5$ .

Before proceeding, we have to check that these identifications are consistent. First, for both  $e_1$  and  $e_2$ , the same spacetime motion is induced by initially translating and then rotating as by initially rotating and then performing an inverse translation. The two sequences of operations must yield the same net transformation on the fields. Since the rotation is gauge trivial, we require  $T_1 = T_1^{-1}$  and  $T_2 = T_2^{-1}$ , which are satisfied by  $(T_1, T_2) = (T_{51}, T_{5'1'})$ . The other consistency condition arises because performing an  $e_1$  translation followed by an  $e_2$  translation induces the same spacetime motion as does  $e_2$  followed by  $e_1$ . Hence, we require  $[T_1, T_2] = 0$ , which is also clearly satisfied.

### 3.2 Gauge symmetries at the orbifold fixed points

At special points on the orbifold, certain gauge transformation parameters are forced to vanish. Therefore, the matter content and interactions located on the fixed point need only respect the gauge symmetries whose transformation parameters are non-vanishing there [6, 13]. Consider, for instance, the  $z = \pi R_5$  fixed point. We have  $V(z = \pi R_5) = V(z = -\pi R_5)$  from the  $Z_2$  rotation and  $V(z = \pi R_5) = T_1 V(z = -\pi R_5) T_1^{-1}$  from the  $e_1$  translation. These equations are consistent only if the wavefunction of every non- $SU(5) \otimes U(1)_X$  gauge field, and every non- $SU(5) \otimes U(1)_X$  gauge transformation parameter, vanishes at the  $z = \pi R_5$  fixed point. Thus, interactions at this point need only preserve  $SU(5) \otimes U(1)_X$ . Similarly, interactions at  $z = \pi i R_6$  need only preserve  $SU(5)' \otimes U(1)'_X$ . For the  $z = \pi(R_5 + iR_6)$  fixed point, one must apply both  $e_1$  and  $e_2$  translations to compare with the result from performing the  $Z_2$  rotation, and one finds  $V(z = \pi(R_5 + iR_6)) = T_1 T_2 V(z = -\pi(R_1 + iR_2)) T_2^{-1} T_1^{-1}$ . Since  $T_1 T_2 = \sigma_0 \otimes \text{diag}(1, 1, 1, -1, -1)$ , which commutes only with the generators listed in Eq. (3), one learns that interactions at this fixed point need only preserve  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ . Finally, at the fixed point at the origin,



$z$	gauge symmetry
0	$SO(10)$
$\pi R_5$	$SU(5) \otimes U(1)_X$
$\pi i R_6$	$SU(5)' \otimes U(1)'_X$
$\pi(R_5 + i R_6)$	$SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$

Table 1: Supersymmetry and gauge symmetry on each of the four fixed points.

there is no restriction on the gauge transformation parameters, so that this point preserves the full  $SO(10)$ . The non-trivial gauge symmetries acting at each fixed point are shown in Table 1. At each fixed point, the 4D  $N = 1$  supersymmetry is preserved.

The appearance of the residual  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$  gauge symmetry at the  $z = \pi(R_5 + i R_6)$  fixed point suggests that there is a modified version of this model that is equally viable. Instead of taking  $T_2 = T_{5'1'}$ , so that  $z = \pi i R_6$  is an  $SU(5)' \otimes U(1)'_X$  preserving fixed point, one could instead choose  $T_2 = T_{422}$ . Now the  $z = \pi i R_6$  fixed point preserves  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ , while the  $z = \pi(R_5 + i R_6)$  fixed point preserves  $SU(5)' \otimes U(1)'_X$ . The unbroken gauge group is  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$  as before. Note that  $T_{422}$  is a  $T_{3R}$  rotation by angle  $\pi$ , so that  $SU(5)$  and flipped  $SU(5)$  are related by a  $T_{3R}$  flip.

### 3.3 Doublet-triplet splitting

One attractive feature of GUT breaking on orbifolds is that natural doublet-triplet splitting may occur if the Higgs multiplets propagate in the bulk [16], and it does occur naturally in the present model. Consider a hypermultiplet  $\mathbf{H}_{10} = (H_{10}, H_{10}^c)$ . The form of the Lagrangian forces us to assign opposite parities to  $H_{10}$  and  $H_{10}^c$  under the  $Z_2$  rotation. Without loss of generality, we take  $H_{10}(+)$  and  $H_{10}^c(-)$ , so that  $H_{10}^c(z = 0)$  is forced to vanish, and only  $H_{10}$  can contain massless zero modes. How does  $H_{10}$  transform under the  $e_1$  and  $e_2$  translations? For the action to be invariant, we need

$$H_{10}(z + e_i) = P_i T_i H_{10}(z). \quad (16)$$

Here  $P_i$  are  $\pm 1$ : invariance of the action allows these to be arbitrary phases, but consistency of the transformation properties of  $H_{10}$  requires  $(P_i T_i)^{-1} = P_i T_i$ .

Under  $SU(5)$ ,  $H_{10}$  decomposes as  $H_5 + H_{\bar{5}}$ . In terms of our previous notation, the  $SU(5)$  generators for these representations come from  $S_5 + A_5$  and  $S_5 - A_5$ , respectively. Referring to Eq. (1), we conclude that the  $H_5$  and  $H_{\bar{5}}$  contained in  $H_{10}$  are eigenvectors of  $\sigma_2 \otimes I_5$  with eigenvalues  $+1$  and  $-1$ , respectively. Hence the components of  $H_{10}$  have parities under the  $e_1$

and  $e_2$  translations given by

$$H_{\mathbf{10}} = \begin{pmatrix} h_3(P_1, P_2) \\ h_2(P_1, -P_2) \\ \bar{h}_3(-P_1, -P_2) \\ \bar{h}_2(-P_1, P_2) \end{pmatrix}, \quad (17)$$

where  $H_{\mathbf{5}} = (h_3, h_2)$  and  $H_{\bar{\mathbf{5}}} = (\bar{h}_3, \bar{h}_2)$ . (For flipped  $SU(5)$ , the alternative association of doublets with triplets is made  $H_{\mathbf{5}'} = (h_3, \bar{h}_2)$  and  $H_{\bar{\mathbf{5}}'} = (\bar{h}_3, h_2)$ .) No matter what choices are made for  $P_i$ , only one of  $h_3, \bar{h}_3, h_2$  and  $\bar{h}_2$  has a zero mode. As in the  $SU(5)$  case [4], doublet-triplet splitting is a necessary consequence of the orbifold gauge symmetry breaking.

Notice that a single  $\mathbf{H}_{\mathbf{10}}$  hypermultiplet in the bulk leads to a low energy theory that is anomalous under the unbroken gauge group. It is necessary to introduce combinations of bulk hypermultiplets such that the collection of zero modes is anomaly free (bulk anomalies are discussed in the following subsection). Here we restrict hypermultiplets to be of low dimension, either  $\mathbf{10}$  or  $\mathbf{16}$ . With this restriction it is interesting that there are only two combinations involving a  $\mathbf{10}$  which give vanishing 4D anomaly:  $\mathbf{H}_{\mathbf{10}}(P_1, P_2)$  and  $\mathbf{H}'_{\mathbf{10}}(-P_1, -P_2)$ , with  $P_i$  of the same (opposite) signs, leading to zero mode triplets (doublets)

$$\begin{aligned} H_{\mathbf{10}}(+, +) + H'_{\mathbf{10}}(-, -) &\rightarrow h_3 + \bar{h}_3, \\ H_{\mathbf{10}}(+, -) + H'_{\mathbf{10}}(-, +) &\rightarrow h_2 + \bar{h}_2. \end{aligned} \quad (18)$$

Since  $\mathbf{H}_{\mathbf{10}}$  and  $\mathbf{H}'_{\mathbf{10}}$  must have the same 6D chirality, these fields cannot have a bulk mass term. However, they can have mass term localized on 4D fixed points. Here we assume a vanishing brane mass term between  $\mathbf{H}_{\mathbf{10}}$  and  $\mathbf{H}'_{\mathbf{10}}$ .

Starting from a ten dimensional hypermultiplet placed in the bulk, the orbifold breaking of  $SO(10) \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$  requires an additional ten dimensional hypermultiplet to cancel 4D anomalies. Therefore, doublet-triplet splitting is a necessary consequence of the orbifold breaking, assuming that brane localized mass terms are absent. To negate it would require adding both combinations of Eq. (18). Thus, we can identify the two Higgs doublets as the smallest set of hypermultiplets whose zero modes yield vanishing 4D anomaly.

### 3.4 Bulk anomalies

For the present theory to be consistent, we need cancellation of the anomalies both on the 4D fixed point, which are calculated by considering the 4D anomaly of the zero modes, and in the 6D bulk [12]. Interestingly, the 6D irreducible gauge anomalies do cancel in the present model with a vector multiplet and two  $\mathbf{10}$  hypermultiplets  $\mathbf{H}_{\mathbf{10}}$  and  $\mathbf{H}'_{\mathbf{10}}$  in the bulk [12]. Moreover, the irreducible gauge anomaly from a  $\mathbf{10}$  is equal and opposite to that of a  $\mathbf{16}$  (or a  $\overline{\mathbf{16}}$ ) with

the same 6D chirality, so this cancellation is maintained provided that each additional **10** comes with either a **16** or a  $\overline{\mathbf{16}}$ .

Unfortunately, the mixed gauge-gravitational anomalies do not cancel, so that the present  $T^2/Z_2$  model is in fact inconsistent in its present form. This is a major motivation for considering the anomaly-free models of sections 4 and 5. However, there may be possible that both the irreducible gauge and mixed gauge-gravitational anomalies are canceled by introducing extra  $SO(10)$  multiplets in the bulk whose zero modes become heavy by having mass terms with brane localized fields. In the rest of this section, we put this anomaly problem aside, and continue exploring the  $T^2/Z_2$  model, requiring only that the irreducible gauge anomalies cancel.

### 3.5 Quarks and leptons in the bulk

Where should the standard model quarks and leptons be located in this model? We first consider the possibility that they appear in hypermultiplet **16**s that propagate in the bulk. As discussed above, we are then led to introduce an extra **10** for each **16** to cancel irreducible gauge anomalies. We assume that these extra **10**s pair up to become heavy through large brane localized mass terms (although the **10**s containing the Higgs doublets must not obtain such a mass term).

#### 3.5.1 Minimal matter content from anomaly cancellation

Imagine a hypermultiplet  $\Psi_{\mathbf{16}} = (\psi_{\mathbf{16}}, \psi_{\overline{\mathbf{16}}}^c)$  propagating in the bulk. We assign the parities under the  $Z_2$  rotation as  $(+, -)$  so that the conjugate matter does not have a zero mode. The transformation properties of  $\psi_{\mathbf{16}}$  under the torus translations are most easily deduced by considering the  $SU(5) \otimes U(1)_X$  decomposition of the matter couplings to gauge fields. The gauge fields decompose as  $\mathbf{45} = \mathbf{24}_0 + \mathbf{10}_4 + \overline{\mathbf{10}}_{-4} + \mathbf{1}_0$ , and the matter decomposes as  $\mathbf{16} = \mathbf{10}_{-1} + \overline{\mathbf{5}}_3 + \mathbf{1}_{-5}$ . Since we know that the  $\mathbf{10}_4$  and  $\overline{\mathbf{10}}_{-4}$  gauge fields have negative parity under the  $e_1$  translation, the existence of the  $\psi_{\overline{\mathbf{5}}}^\dagger V_{\mathbf{10}} \psi_{\mathbf{10}}$  and  $\psi_{\mathbf{10}}^\dagger V_{\mathbf{10}} \psi_{\mathbf{1}}$  terms in the Lagrangian implies that we can take the parities for  $(\psi_{\mathbf{10}}, \psi_{\overline{\mathbf{5}}}, \psi_{\mathbf{1}})$  to be either  $(-, +, +)$  or  $(+, -, -)$  ( $U(1)_X$  charges omitted for notational simplicity). Similarly, under the  $e_2$  translation one can take the parities for  $(\psi_{\mathbf{10}'}, \psi_{\overline{\mathbf{5}}'}, \psi_{\mathbf{1}'})$  to be either  $(-, +, +)$  or  $(+, -, -)$ . Here  $\mathbf{10}'_{-1}$ ,  $\overline{\mathbf{5}}'_3$  and  $\mathbf{1}'_{-5}$  denote the decomposition of **16** under  $SU(5)'$ , or flipped  $SU(5)$ , which means that  $\psi_{\mathbf{10}'} = (Q, D, N)$ ,  $\psi_{\overline{\mathbf{5}}'} = (U, L)$  and  $\psi_{\mathbf{1}'} = E$ . Overall, there are four choices for the parities:

$$\begin{aligned}
\mathbf{16}_{++} & \quad (\text{zero mode}) = Q, \\
\mathbf{16}_{+-} & \quad (\text{zero modes}) = U, E, \\
\mathbf{16}_{-+} & \quad (\text{zero modes}) = D, N, \\
\mathbf{16}_{--} & \quad (\text{zero mode}) = L,
\end{aligned} \tag{19}$$

where the first  $\pm$  sign refers to the parity of  $\mathbf{10}_{-1}$  under the  $e_1$  translation, while the second refers to the parity of  $\mathbf{10}'_{-1}$  under the  $e_2$ . Thus, for each generation of matter that propagates in the bulk, we need four hypermultiplets whose parities under the torus translations conspire to yield a complete  $SO(10)$  multiplet for the massless zero modes.

The conspiracy of parities required to obtain a complete generation of massless zero modes is required by cancellation of zero mode anomalies. In subsection 3.3, we saw that the smallest anomaly-free set came from two  $\mathbf{10}$ s, forming two Higgs doublets. We now show that the smallest anomaly-free set of hypermultiplets with *chiral* zero modes under  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$  is the combination of four  $\mathbf{16}$ s described above, forming a single generation of matter including the right-handed neutrino.

The smallest representation of  $SO(10)$  is the vector,  $\mathbf{10}$ , but since this representation is real, we cannot obtain chiral matter from any combination of  $\mathbf{10}$ s. The next simplest possibility is to try some combination of  $\mathbf{10}$ s and  $\mathbf{16}$ s. If there were no  $U(1)_X$ , the smallest anomaly free combination which is chiral would be  $\mathbf{10}_{-+}$ ,  $\mathbf{10}_{--}$ ,  $\mathbf{16}_{++}$  and  $\mathbf{16}_{+-}$ , leaving  $L$ ,  $D$ ,  $Q$  and  $\{U, E\}$ , respectively, in zero modes. However, this set is anomalous under  $U(1)_X$ . Remember that coming from  $\mathbf{10}$ s,  $L$  and  $D$  have a ‘wrong’  $U(1)_X$  quantum number of  $-2$ . The simplest potential cure would be to add the right-handed neutrino  $N$ . Note that  $\mathbf{10}$  cannot give  $N$ , so that we must use  $\mathbf{16}_{-+}$ , which gives  $D$  and  $N$ , instead of  $\mathbf{10}_{--}$ . Even then, however, the wrong  $U(1)_X$  charge of  $L$  from  $\mathbf{10}_{-+}$  still fails to cancel anomaly associated with  $U(1)_X$ , forcing us to give up  $\mathbf{10}_{-+}$  and to use  $\mathbf{16}_{--}$ .

Therefore, although four  $\mathbf{16}$  hypermultiplets are needed to complete a single generation, it does not spoil much the unification of matter as a virtue of underlying  $SO(10)$ . One might think that if we identify matter as the smallest set of hypermultiplets giving anomaly-free chiral zero modes, then  $SU(5)$  can also explain the matter quantum numbers. In the  $SO(10)$  case, however, there is naturally a  $U(1)_X$  gauge symmetry, so that it requires the right-handed neutrino  $N$  to be present in a generation.

The matter fields can couple to the Higgs fields on any one of the four fixed points. For instance, on the  $SO(10)$  preserving brane we have the Yukawa couplings

$$S = \int d^6x \delta^2(z) \left\{ \int d^2\theta (\lambda \psi_{\mathbf{16}} \psi_{\mathbf{16}} H_{\mathbf{10}} + \lambda' \psi_{\mathbf{16}} \psi_{\mathbf{16}} H'_{\mathbf{10}}) + \text{h.c.} \right\}, \quad (20)$$

where we have suppressed indices labeling the various  $\psi_{\mathbf{16}}$ ’s. Note that there are no GUT fermion mass relations in the present setup because the massless zero modes that comprise a single generation originate from different  $\mathbf{16}$ s.

### 3.5.2 Supersymmetry breaking, $U(1)_X$ breaking, and neutrino masses

What is an appropriate mechanism for breaking the remaining 4D  $N = 1$  supersymmetry? Gaugino mediation is not accommodated by the present model because matter propagates in the

bulk and cannot be spatially separated from the supersymmetry breaking. A different, feasible mechanism, which we can adopt here, is given by the type of Scherk-Schwarz supersymmetry breaking described in Ref. [8], involving a small parameter that lowers the breaking scale relative to the compactification scale.

Two other phenomenological questions concern the unbroken  $U(1)_X$  and the origin of masses for the right-handed neutrinos. We could try to break  $U(1)_X$  by more complicated orbifolds, where parity operations become noncommutative, but we will not investigate this possibility here. Instead, we break  $U(1)_X$  by driving a GUT-scale vacuum expectation value (VEV) for a field  $X$  transforming as a singlet under  $SU(5)$  but with charge 10 under  $U(1)_X$ . (The  $\overline{X}$  field with the opposite  $U(1)_X$  charge is also introduced to cancel anomalies.) This VEV can also be used to give a large mass to the right-handed neutrinos. Since the field is an  $SU(5)$  singlet, we must localize it to the  $SU(5) \otimes U(1)_X$  brane. The interaction term responsible for  $U(1)_X$  breaking is

$$S = \int d^6x \delta^2(z - \pi R_5) \left\{ \int d^2\theta k X N_\psi N_\psi + \text{h.c.} \right\}, \quad (21)$$

where  $N_\psi$  is the right-handed neutrino coming from the bulk hypermultiplet  $\psi_{\mathbf{16}}$ .

If Scherk-Schwarz supersymmetry breaking is employed as in Ref. [8], it gives soft supersymmetry breaking masses of the order the weak scale to  $N_\psi$ , while  $X$  remains massless, since it is located on the brane. Therefore, the above interaction drives the mass-squared of  $X$  scalar negative, while that of  $N_\psi$  remains positive. Since there is no large quartic potential in the flat direction  $X = \overline{X}$ , we have a runaway situation and obtain a huge VEV for  $X$  (presumably around the compactification scale), which breaks  $U(1)_X$  at very high energy scale.

After this breaking, the right-handed neutrinos receive large Majorana masses from Eq. (21). In order to get the right order of magnitude for the neutrino masses through the see-saw mechanism, the  $k$  couplings must be somewhat small of order  $10^{-2} - 10^{-3}$ .

### 3.6 Quarks and leptons on branes

Now we consider an alternative possibility that the quarks and leptons are localized to one of the fixed points. If they live on the  $z = 0$  fixed point the quarks and leptons are forced to appear in full  $SO(10)$  multiplets, since the full  $SO(10)$  gauge symmetry is realized there. We thus introduce three  $\psi_{\mathbf{16}}$ 's each transforming as  $\mathbf{16}$  under  $SO(10)$ . This setup provides the same understanding of the standard-model fermion quantum numbers as is given by the standard 4D  $SO(10)$ .

Brane-localized interactions of the  $\psi_{\mathbf{16}}$ 's with the bulk Higgs multiplets give rise to Yukawa couplings

$$S = \int d^6x \delta^2(z) \left\{ \int d^2\theta (\lambda \psi_{\mathbf{16}} \psi_{\mathbf{16}} H_{\mathbf{10}} + \lambda' \psi_{\mathbf{16}} \psi_{\mathbf{16}} H'_{\mathbf{10}}) + \text{h.c.} \right\}. \quad (22)$$

Since the up- and down-type Higgs doublets come from different  $SO(10)$  multiplets, the fermion mass relations are those of  $SU(5)$  rather than those of  $SO(10)$ . Realistic fermion masses may be obtained through mixing of the  $\psi_{\mathbf{16}}$ 's on the fixed point with bulk  $\mathbf{16}$ s, in a way similar to what was done for fermion masses in the  $SU(5)$  case in Ref. [6]. Again, for cancellation of irreducible gauge anomalies, these bulk  $\mathbf{16}$ s must be accompanied by  $\mathbf{10}$  hypermultiplets, which must obtain large brane localized mass terms.

If matter is localized to one of the other fixed points, its Yukawa interactions will only respect the reduced gauge symmetry remaining at the fixed point. On the 5-1 and 4-2-2 branes, these interactions give rise to  $SU(5)$  fermion mass relations, which again may be corrected through mixing with bulk states. On the 5'-1' fixed point, on the other hand, there are no GUT relations for the fermion masses, other than equality between the up-type quark and neutrino Dirac mass matrices.

With matter localized on one of the fixed points, gaugino mediation of supersymmetry breaking [22] is easily accommodated, provided that supersymmetry breaking occurs on a different fixed point from the one where matter resides. Gaugino mass relations at the compactification scale depend on which is the supersymmetry breaking fixed point: we have  $M_3 = M_2 = M_1$  if supersymmetry is broken on the 10 or 5-1 branes, and  $M_3 = M_2 \neq M_1$  if supersymmetry is broken on the 4-2-2 or 5'-1' branes.

As in the bulk matter case, an obvious location for  $U(1)_X$  breaking is the 5-1 brane. However, unless matter is localized to this brane, it does not feel the breaking directly, so that communication through exchanges of bulk states is required. Consider, for example, the case where matter lives on the 10 brane. Suppose that bulk states  $\chi_{\mathbf{16}} + \bar{\chi}_{\mathbf{16}}$  with a brane mass term couple to the 5-1 and 10 branes according to

$$S = \int d^2\theta \left\{ \delta^2(z - \pi R_5) \left( Y(X\bar{X} - \mu^2) + X N_{\bar{\chi}} N_{\bar{\chi}} + \bar{X} \bar{N}_{\chi} \bar{N}_{\chi} \right) + \delta^2(z) \bar{\chi}_{\mathbf{16}} \bar{\chi}_{\mathbf{16}} \psi_{\mathbf{16}} \psi_{\mathbf{16}} \right\} + \text{h.c.} \quad (23)$$

Here we have neglected coupling constants,  $Y$  is a singlet superfield, and  $X$  and  $\bar{X}$  are  $SU(5)$  singlets with  $U(1)_X$  charges 10 and  $-10$ , respectively.  $N_{\bar{\chi}}$  represents “right-handed neutrino” components in  $\bar{\chi}_{\mathbf{16}}$  and similarly for  $\bar{N}_{\chi}$ . The first term in the above interaction forces  $X$  and  $\bar{X}$  to acquire VEVs equal to  $\mu$ . Upon integrating out  $N_{\bar{\chi}}$  and  $\bar{N}_{\chi}$  states, the non-local term  $X N_{\psi} N_{\psi}$  is generated, giving rise to masses for the right-handed neutrinos  $N_{\psi}$ . This term carries a suppression by powers of  $(M_{\chi} R)$  if the brane mass term  $M_{\chi}$  for  $\chi_{\mathbf{16}} + \bar{\chi}_{\mathbf{16}}$  is localized on  $z = 0$  or  $\pi R_5$ , since the wavefunctions for  $\chi_{\mathbf{16}}$  and  $\bar{\chi}_{\mathbf{16}}$  are suppressed there. This may give the correct order of magnitude for the right-handed neutrino masses.

## 4 A Model on $T^2/Z_6$

### 4.1 Orbifold structure

As discussed in subsection 3.4, the  $T^2/Z_2$  model is anomalous. We are thus led to consider a different setup for  $SO(10)$  in 6D. In particular, we take the bulk to have 6D  $N = 2$  supersymmetry rather than  $N = 1$ , so that there are no bulk anomalies. The  $N = 2$  supersymmetry in 6D corresponds to  $N = 4$  supersymmetry in 4D, so that only the gauge multiplet can be introduced in the bulk. This multiplet can be decomposed under a 4D  $N = 1$  supersymmetry into a vector multiplet  $V$  and three chiral multiplets  $\Sigma$ ,  $\Phi$ , and  $\Phi^c$  in the adjoint representation, with the fifth and sixth components of the gauge field,  $A_5$  and  $A_6$ , contained in the lowest component of  $\Sigma$ .

Using 4D  $N = 1$  language, the bulk action may be written as [21]

$$S = \int d^6x \left\{ \text{Tr} \left[ \int d^2\theta \left( \frac{1}{4kg^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \frac{1}{kg^2} \left( \Phi^c \partial \Phi - \frac{1}{\sqrt{2}} \Sigma [\Phi, \Phi^c] \right) \right) + \text{h.c.} \right] \right. \\ \left. + \int d^4\theta \frac{1}{kg^2} \text{Tr} \left[ (\sqrt{2} \partial^\dagger + \Sigma^\dagger) e^{-V} (-\sqrt{2} \partial + \Sigma) e^V + \Phi^\dagger e^{-V} \Phi e^V + \Phi^{c\dagger} e^{-V} \Phi^c e^V \right] \right\}, \quad (24)$$

in the Wess-Zumino gauge.

Can we build a realistic model starting with 6D  $N = 2$  supersymmetry on  $T^2/Z_2$ ? The trouble is that for this orbifold the fixed points are left with 4D  $N = 2$  supersymmetry rather than 4D  $N = 1$  supersymmetry. To reduce the supersymmetry further requires an orbifold in which more modding out is done — a fairly simple orbifold that works is  $T^2/Z_6$ . This orbifold is constructed by identifying points of the infinite plane  $R^2$  under three operations,  $\mathcal{Z} : z \rightarrow \omega z$ ,  $\mathcal{T}_1 : z \rightarrow z + 2\pi R$  and  $\mathcal{T}_2 : z \rightarrow z + 2\pi\omega R$ , where  $\omega = e^{i\pi/3}$ . The identifications for the fields under  $\mathcal{Z}$  are taken to be

$$V(\omega z) = T_{422} V(z) T_{422}^{-1}, \quad (25)$$

$$\Sigma(\omega z) = \omega^5 T_{422} \Sigma(z) T_{422}^{-1}, \quad (26)$$

$$\Phi(\omega z) = \omega^5 T_{422} \Phi(z) T_{422}^{-1}, \quad (27)$$

$$\Phi^c(\omega z) = \omega^2 T_{422} \Phi^c(z) T_{422}^{-1}, \quad (28)$$

and the identifications under  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are

$$V(z + 2\pi R) = T_{51} V(z) T_{51}^{-1}, \quad (29)$$

$$\Sigma(z + 2\pi R) = T_{51} \Sigma(z) T_{51}^{-1}, \quad (30)$$

$$\Phi(z + 2\pi R) = T_{51} \Phi(z) T_{51}^{-1}, \quad (31)$$

$$\Phi^c(z + 2\pi R) = T_{51} \Phi^c(z) T_{51}^{-1}, \quad (32)$$

and

$$V(z + 2\pi\omega R) = T_{51}V(z)T_{51}^{-1}, \quad (33)$$

$$\Sigma(z + 2\pi\omega R) = T_{51}\Sigma(z)T_{51}^{-1}, \quad (34)$$

$$\Phi(z + 2\pi\omega R) = T_{51}\Phi(z)T_{51}^{-1}, \quad (35)$$

$$\Phi^c(z + 2\pi\omega R) = T_{51}\Phi^c(z)T_{51}^{-1}, \quad (36)$$

respectively. This choice of identifications breaks the  $SO(10)$  gauge group to  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$  at low energies, and the only massless zero modes are those of  $V$ . This orbifold has a single fixed point located at  $z = 0$ , which has 4D  $N = 1$  supersymmetry and 4-2-2 gauge symmetry. However, there is another special point that is fixed under the  $Z_3$  subgroup of  $Z_6$ , located at  $z = (2\pi R/\sqrt{3})e^{i\pi/6}$ . This point has 5-1 gauge symmetry, and also has only 4D  $N = 1$  supersymmetry.

## 4.2 Matter configuration

There are only two possibilities for where the quarks and leptons live in this model, corresponding to the 4-2-2 and 5-1 points. Higgs multiplets are not allowed to propagate in the bulk because of the 6D  $N = 2$  supersymmetry, so we are forced to put the Higgs on the same brane as the quarks and leptons. If we choose the 5-1 brane, we are faced with a difficult doublet-triplet splitting problem, as the Higgs doublets must appear in  $\mathbf{5}_2 + \overline{\mathbf{5}}_{-2}$  multiplets under  $SU(5) \otimes U(1)_X$ , and the colored components must somehow get heavy. We thus focus on the alternative placement on the 4-2-2 brane. On this point a generation of matter is formed from the  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$  multiplets  $(\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ , and the Higgs doublets of the MSSM are contained in a  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  multiplet. Gaugino mediation is naturally realized in this model by localizing the breaking of 4D  $N = 1$  supersymmetry to the 5-1 brane.

## 4.3 $U(1)_X$ breaking and fermion masses

The  $U(1)_X$  is easily broken on the 5-1 brane by introducing a brane-localized  $SU(5)$  singlet  $X$  charged under  $U(1)_X$ , and a superpotential that forces it to acquire a VEV. However, the question of how this  $U(1)_X$  breaking is communicated to the 4-2-2 brane to give rise to Majorana masses for the right-handed neutrinos is not straightforward. Unfortunately, the states contained in the adjoint of  $SO(10)$  do not have the correct quantum numbers to generate  $XNN$  as a non-local operator, and the 6D  $N = 2$  supersymmetry prevents us from adding additional bulk states. A problem related to that of the right-handed neutrino masses is that this model has  $SO(10)$  fermion mass relations: somehow, the 4-2-2 brane must be made to feel 4-2-2 breaking.

Right-handed neutrino masses could be generated if a multiplet  $X'$  on the 4-2-2 brane acquired a VEV that broke  $U(1)_X$  but not the standard model gauge group. In this case, however, there is



a vacuum alignment problem because the potential for  $X'$  is 4-2-2 symmetric. Correspondingly, a potential that forces  $X'$  to take on a VEV will lead to extra massless Goldstone states. For instance, such a VEV cannot be  $SU(2)_R$  globally symmetric, and there are no  $SU(2)_R$  gauge bosons to eat the Goldstones, as they are already made heavy by the orbifold compactification.

Changes to this picture come from radiative corrections to the potential for  $X'$  below the compactification scale. If the theory were not supersymmetric these corrections would give masses of order  $\alpha v/(4\pi)$  to the Goldstones, where  $v$  is the VEV of  $X'$ . However, in the supersymmetric limit the Goldstones pick up no mass, and they thus only acquire TeV-scale masses from gaugino mediation, just as do the squarks and sleptons. Whether it is a realistic possibility that these corrections to the potential force  $X'$  to point in an appropriate direction is a question we leave for future study. In any case this setup reveals a crucial point: in theories where a gauge generator is broken both by the orbifold projection and by a brane VEV, there will be a corresponding “would-be Goldstone” with mass  $\sim m_{\text{SUSY}}$ . These states generically spoil the success of the gauge coupling unification in the MSSM, and might be problematic for proton stability. This is independent of  $1/R$  and the scale of the brane VEV. There is such a TeV supermultiplet for each generator which is “broken twice”.

## 5 A Model on $T^2/(Z_2 \times Z'_2)$

In the  $T^2/Z_2$  model of section 3, the  $U(1)_X$  left over after orbifolding was broken by the VEV of a field transforming under  $U(1)_X$  only. In contrast, in the model of section 4 either we are left with no Majorana masses for the righthanded neutrinos, or we must require a field  $X'$  to have additional gauge transformation properties and a vacuum alignment problem must be resolved to ensure that the standard model gauge group remains unbroken at low energies. In this section we construct a third model, on a  $T^2/(Z_2 \times Z'_2)$  orbifold, in which both the vacuum alignment problem of the  $T^2/Z_6$  model, as well as the anomalies of the  $T^2/Z_2$  model, are absent.

### 5.1 Orbifold structure

We again consider a theory with 6D  $N = 2$  supersymmetry. Using the 4D  $N = 1$  language, we can express the bulk action as [21]

$$\begin{aligned}
S = & \int d^6x \left\{ \text{Tr} \left[ \int d^2\theta \left( \frac{1}{4kg^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \frac{1}{kg^2} \left( \Phi \partial_5 \Sigma_6 - \Phi \partial_6 \Sigma_5 - \frac{1}{\sqrt{2}} \Phi [\Sigma_5, \Sigma_6] \right) \right) + \text{h.c.} \right] \right. \\
& + \int d^4\theta \frac{1}{kg^2} \text{Tr} \left[ (\sqrt{2} \partial_5 + \Sigma_5^\dagger) e^{-V} (-\sqrt{2} \partial_5 + \Sigma_5) e^V + (\sqrt{2} \partial_6 + \Sigma_6^\dagger) e^{-V} (-\sqrt{2} \partial_6 + \Sigma_6) e^V \right. \\
& \left. \left. + \Phi^\dagger e^{-V} \Phi e^V + \partial_5 e^{-V} \partial_5 e^V + \partial_6 e^{-V} \partial_6 e^V \right] \right\}, \tag{37}
\end{aligned}$$

in the Wess-Zumino gauge. When expressed in terms of components, this action and that of Eq. (24) have identical forms. The orbifold of the present model will preserve a different 4D  $N = 1$  supersymmetry than the orbifold of the previous one (namely, one in which  $A_5$  and  $A_6$  appear in different superfields), and we have chosen to make this different 4D  $N = 1$  supersymmetry manifest.

The orbifold  $T^2/(Z_2 \times Z'_2)$  is constructed by identifying points of the infinite plane  $R^2$  under four operations,  $\mathcal{Z}_1 : (x^5, x^6) \rightarrow (-x^5, x^6)$ ,  $\mathcal{Z}_2 : (x^5, x^6) \rightarrow (x^5, -x^6)$ ,  $\mathcal{T}_1 : (x^5, x^6) \rightarrow (x^5 + 2\pi R_5, x^6)$  and  $\mathcal{T}_2 : (x^5, x^6) \rightarrow (x^5, x^6 + 2\pi R_6)$ . Here, for simplicity, we have taken the two translations  $\mathcal{T}_1$  and  $\mathcal{T}_2$  to be in orthogonal directions.

Under  $\mathcal{Z}_1$  and  $\mathcal{Z}_2$  we make the gauge-trivial identifications

$$V(-x^5, x^6) = V(x^5, x^6), \quad (38)$$

$$\Sigma_5(-x^5, x^6) = -\Sigma_5(x^5, x^6), \quad (39)$$

$$\Sigma_6(-x^5, x^6) = \Sigma_6(x^5, x^6), \quad (40)$$

$$\Phi(-x^5, x^6) = -\Phi(x^5, x^6), \quad (41)$$

and

$$V(x^5, -x^6) = V(x^5, x^6), \quad (42)$$

$$\Sigma_5(x^5, -x^6) = \Sigma_5(x^5, x^6), \quad (43)$$

$$\Sigma_6(x^5, -x^6) = -\Sigma_6(x^5, x^6), \quad (44)$$

$$\Phi(x^5, -x^6) = -\Phi(x^5, x^6), \quad (45)$$

respectively. Note that various signs appearing in Eqs. (38 – 45) are determined by invariance of the bulk action under the  $\mathcal{Z}_{1,2}$  operations.

The  $\mathcal{Z}_1$  identification breaks 4D  $N = 4$  supersymmetry to 4D  $N = 2$  supersymmetry (or equivalently, 6D  $N = 2$  to 6D  $N = 1$  supersymmetry), with  $(V, \Sigma_6)$  forming a vector multiplet and  $(\Sigma_5, \Phi)$  forming a hypermultiplet. Similarly, the  $\mathcal{Z}_2$  identification breaks 4D  $N = 4$  supersymmetry to 4D  $N = 2$  supersymmetry, with  $(V, \Sigma_5)$  forming a vector multiplet and  $(\Sigma_6, \Phi)$  forming a hypermultiplet. This means that the two  $N = 2$  supersymmetries remaining after the  $\mathcal{Z}_1$  and  $\mathcal{Z}_2$  operations are different subgroups of the original  $N = 4$  supersymmetry. Thus, the combination of  $\mathcal{Z}_1$  and  $\mathcal{Z}_2$  identifications, *i.e.* the  $T^2/(Z_2 \times Z'_2)$  compactification, breaks the original 6D  $N = 2$  supersymmetry all the way down to 4D  $N = 1$  supersymmetry.

The  $\mathcal{T}_1$  and  $\mathcal{T}_2$  identifications are

$$V(x^5 + 2\pi R_5, x^6) = T_{51} V(x^5, x^6) T_{51}^{-1}, \quad (46)$$

$$\Sigma_5(x^5 + 2\pi R_5, x^6) = T_{51} \Sigma_5(x^5, x^6) T_{51}^{-1}, \quad (47)$$

$$\Sigma_6(x^5 + 2\pi R_5, x^6) = T_{51} \Sigma_6(x^5, x^6) T_{51}^{-1}, \quad (48)$$

$$\Phi(x^5 + 2\pi R_5, x^6) = T_{51} \Phi(x^5, x^6) T_{51}^{-1}, \quad (49)$$

$(x^5, x^6)$	4D supersymmetry	gauge symmetry
$(0, 0)$	$N = 1$	$SO(10)$
$(\pi R_5, 0)$	$N = 1$	$SU(5) \otimes U(1)_X$
$(0, \pi R_6)$	$N = 1$	$SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$
$(\pi R_5, \pi R_6)$	$N = 1$	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$

Table 2: Supersymmetry and gauge symmetry on each of the four fixed points.

fixed lines	4D supersymmetry	gauge symmetry
$x^5 = 0$	$N = 2$	$SO(10)$
$x^6 = 0$	$N = 2$	$SO(10)$
$x^5 = \pi R_5$	$N = 2$	$SU(5) \otimes U(1)_X$
$x^6 = \pi R_6$	$N = 2$	$SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$

Table 3: Supersymmetry and gauge symmetry on each of the four fixed lines.

and

$$V(x^5, x^6 + 2\pi R_6) = T_{422} V(x^5, x^6) T_{422}^{-1}, \quad (50)$$

$$\Sigma_5(x^5, x^6 + 2\pi R_6) = T_{422} \Sigma_5(x^5, x^6) T_{422}^{-1}, \quad (51)$$

$$\Sigma_6(x^5, x^6 + 2\pi R_6) = T_{422} \Sigma_6(x^5, x^6) T_{422}^{-1}, \quad (52)$$

$$\Phi(x^5, x^6 + 2\pi R_6) = T_{422} \Phi(x^5, x^6) T_{422}^{-1}, \quad (53)$$

respectively. These identifications leave the 3-2-1-1 components of  $V$  as the only ones with massless zero modes. (We could chose  $(T_{51}, T_{5'1'})$  or  $(T_{5'1'}, T_{422})$ , instead of  $(T_{51}, T_{422})$ , for  $(\mathcal{T}_1, \mathcal{T}_2)$  operations. All the arguments in the rest of this section can be extended to these cases in a straightforward way.)

The structure of the fixed points can be worked out by considering the profiles of symmetry transformation parameters in the extra dimensions. On each of the four fixed points of the  $T^2/(Z_2 \times Z'_2)$  orbifold, the remaining supersymmetry and gauge symmetry is given in Table 2 — matter multiplets and interactions placed on the fixed points must respect these symmetries. An important feature of this orbifold is that these fixed points are connected by “fixed lines” with reduced supersymmetry and gauge symmetry. These lines are fixed with respect to one of the  $Z_2$  reflections but not the other. The remaining supersymmetry and gauge symmetry for each such line are given in Table 3. Because of the reduced supersymmetry on these lines, we have additional  $(4 + 1)$ -dimensional subspaces on which matter multiplets may be placed, without

giving rise to anomalies of the 6D bulk. The rich fixed point and fixed line structure of this orbifold provides for a multitude of possibilities for matter locations, fermion mass relations and  $U(1)_X$  breaking, some of which we briefly describe in the next subsection.

## 5.2 Matter configurations

The quarks and leptons may reside on any of the four fixed points or on any of the four fixed lines. If they are localized to the fixed points, for example, a single generation arises as a **16** of  $SO(10)$  for the 10 brane, as  $\mathbf{10}_{-1} + \overline{\mathbf{5}}_3 + \mathbf{1}_{-5}$  under  $SU(5) \otimes U(1)_X$  for the 5-1 brane, as  $(\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$  under  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$  for the 4-2-2 brane, and as the matter multiplets of the standard model, with appropriate  $U(1)_X$  charges, on the 3-2-1-1 brane. Wherever matter resides, the Higgs multiplets should live on a fixed line or point that is in contact with the matter in order to give rise to Yukawa couplings.

For the Yukawa couplings to be on either the 4-2-2 or 3-2-1-1 branes the Higgs multiplets can either propagate on a touching fixed line, or they can live on those points as a  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  or  $(\mathbf{1}, \mathbf{2}, 1/2, 2) + (\mathbf{1}, \mathbf{2}, -1/2, -2)$  under the residual 4-2-2 and 3-2-1-1 gauge symmetries, respectively. If they propagate on either the  $x^5 = 0$   $SO(10)$  line or the  $x^5 = \pi R_5$  5-1 line, natural doublet-triplet splitting arises by assigning parities so that the colored triplet zero modes are projected out (this will be illustrated in a specific example shortly). If they propagate on the 4-2-2 line, the colored triplets can be avoided from the start by introducing only  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  multiplets on the line.

If, on the other hand, the Yukawa couplings are on either the 10 or 5-1 branes it is advantageous for the Higgs multiplets to propagate on the  $x^5 = 0$   $SO(10)$  line or the  $x^5 = \pi R_5$  5-1 line, respectively. The reason is that otherwise the Higgs fields are spatially separated from the  $SU(5)$  breaking that arises from orbifolding, making doublet-triplet splitting more problematic.

Gaugino mediated supersymmetry breaking is again easily accommodated by this orbifold, by localizing the supersymmetry breaking to a fixed point from which matter is spatially separated [15]. All three gaugino masses unify if the supersymmetry breaking is on either the 10 or 5-1 points, but there is no unification if the supersymmetry breaking is on either the 4-2-2 or 3-2-1-1 points.

We require  $U(1)_X$  to be broken by the VEV of  $SU(5)$  singlet fields  $X$  and  $\overline{X}$ , with  $U(1)_X$  charges 10 and  $-10$ , which therefore live on either the 5-1 or 3-2-1-1 points. These fields acquire equal VEVs through the brane-localized superpotential  $Y(X\overline{X} - \mu^2)$ . If the matter fields propagate on a line touching the point where  $U(1)_X$  is broken, the right-handed neutrinos obtain masses through the direct superpotential coupling  $XNN$ . Otherwise, the breaking must be communicated by heavy states propagating on the fixed lines.

Clearly, there are numerous interesting theories that may be built on this orbifold. Here we simply consider two simple illustrative examples. Suppose that quarks and leptons are contained

in three  $\psi_{\mathbf{16}}$ 's that live on the  $SO(10)$  fixed point. The best choice for the Higgs multiplets is for them to be contained in a hypermultiplet  $\mathbf{H}_{\mathbf{10}}$  that propagates on the  $x^5 = 0$   $SO(10)$  fixed line. Under the  $\mathcal{Z}_2$  reflection we assign parities  $H_{\mathbf{10}}(+)$  and  $H_{\mathbf{10}}^c(-)$  without loss of generality. Under the 4-2-2 gauge symmetry,  $H_{\mathbf{10}}$  decomposes as  $(\mathbf{1}, \mathbf{2}, \mathbf{2}) + (\mathbf{6}, \mathbf{1}, \mathbf{1})$ , and under the  $\mathcal{T}_2$  translation, these components have opposite parity; with the proper choice of sign only the  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  piece, containing the two Higgs doublets of the MSSM, has a massless zero mode. These massless fields couple to the quarks and leptons through the  $SO(10)$  brane superpotential term  $\psi_{\mathbf{16}}\psi_{\mathbf{16}}H_{\mathbf{10}}$ . Supersymmetry breaking can be localized, for instance, to the 3-2-1-1 brane and mediated by the bulk gauginos. The  $U(1)_X$  breaking can occur on the 5-1 brane and can be communicated to the  $SO(10)$  brane by  $\chi_{\mathbf{16}} + \bar{\chi}_{\mathbf{16}}$  pairs propagating in the  $x^6 = 0$  fixed line as described for the  $T^2/Z_2$  model in subsection 3.6.

One property of this model is that  $SO(10)$  mass relations hold. By mixing the brane-localized  $\psi_{\mathbf{16}}$ 's with  $\mathbf{16}$ s propagating on the  $x^6 = 0$  fixed line using the mechanism of Ref. [6], these relations can be corrected, but  $SU(5)$  mass relations still hold. This remaining  $SU(5)$  mass relations are corrected by further mixing with states on the  $x^5 = 0$  line. A different model with realistic fermion masses is given by starting with the quarks and leptons contained in  $\psi_{\mathbf{16}}$ 's propagating on the  $x^6 = 0$  fixed line. Depending on its parity under  $\mathcal{T}_1$  translations, each  $\psi_{\mathbf{16}}$  contains a zero mode for either  $\bar{\mathbf{5}}_3$  and  $\mathbf{1}_{-5}$  or for  $\mathbf{10}_{-1}$ , where the multiplets are labelled by their transformation properties under  $SU(5) \otimes U(1)_X$ . Cancellation of 4D anomalies then requires these  $\psi_{\mathbf{16}}$ 's to appear in pairs with opposite  $\mathcal{T}_1$  parities, so that each pair yields a full generation. The Higgs multiplets can propagate on either the  $x^5 = 0$  or  $x^5 = \pi R_5$  lines. Taking them to appear in a hypermultiplet  $\mathbf{H}_{\mathbf{10}}$  propagating on the  $x^5 = 0$  line as before, fermion masses again arise from  $\psi_{\mathbf{16}}\psi_{\mathbf{16}}H_{\mathbf{10}}$  superpotential terms localized on the  $SO(10)$  fixed point. This time, the fermion mass relations are those of  $SU(5)$  before mixing with bulk states. These relations can be corrected by mixing components of the  $\psi_{\mathbf{16}}$ 's with states propagating on the the  $x^5 = \pi R_5$  fixed line (which feel the  $SU(5)$  breaking through the  $\mathcal{T}_2$  operation), through couplings on the 5-1 fixed point. Gaugino mediation can be realized in this model by localizing the supersymmetry breaking to the 4-2-2 point. The  $U(1)_X$  breaking can again occur on the 5-1 point, and this time the right-handed neutrinos couple to the breaking directly to pick up their masses.

## 6 Gauge Coupling Unification

In the zero-th order approximation, successful gauge coupling unification is achieved in these models by identifying the compactification scale with the unification scale,  $1/R \sim M_U = 2 \times 10^{16}$  GeV. There are, however, two types of corrections to this naive identification [6, 10].

First, we can write down tree-level gauge kinetic terms that do not respect the full  $SO(10)$  symmetry on subspaces of the 6D spacetime. As an example, we can write 5D gauge kinetic

terms respecting only  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$  gauge symmetry on the  $x^6 = \pi R_6$  fixed line in the  $T^2/(Z_2 \times Z'_2)$  model of section 5. Also, 4D gauge kinetic terms are introduced on each orbifold fixed point, which need only respect the gauge symmetries remaining there. However, the corrections from these operators are generically suppressed by the volume of the extra dimension(s), so that we will neglect these contributions in the following discussion.

The second correction originates from the running of the gauge couplings above the compactification scale due to KK modes. Since the present model is a 6D theory, the zero-mode gauge couplings  $g_{0i}$  at the compactification scale  $M_c$  ( $\equiv 1/R$ ) receive power-law corrections as [23]

$$\frac{1}{g_{0i}^2(M_c)} \simeq \frac{1}{g_0^2(M_*)} - \frac{b}{8\pi^2}((M_*R)^2 - 1) - \frac{b'_i}{8\pi^2}(M_*R - 1) + \frac{b''_i}{8\pi^2} \ln(M_*R), \quad (54)$$

where  $b, b'_i$  and  $b''_i$  are constants of  $O(1)$  and  $M_*$  is the cutoff scale of the theory. In the 6D picture, the last three terms correspond to 6D, 5D and 4D gauge kinetic terms generated by loop effects in the 6D bulk, on the 5D fixed lines and on the 4D fixed points, respectively. An interesting fact is that for the models possessing 6D  $N = 2$  supersymmetry in the bulk, the term quadratically sensitive to the cutoff does not appear,  $b = 0$ . On the other hand, for the  $T^2/Z_2$  model of section 3 the term quadratically sensitive to the cutoff does appear, but the crucial point is that this term is universal, and will not affect the differences between the gauge couplings. In fact, since the bulk  $SO(10)$  gauge symmetry is spoiled only at 4D fixed points, the differential running of the gauge couplings above the compactification scale will be logarithmic, and threshold corrections to  $\sin^2 \theta_w$  will be small.

The same conclusion can be drawn for the  $T^2/Z_6$  model of section 4, which also has  $SO(10)$  gauge symmetry everywhere but on 4D fixed points. The story is different, however, for the  $T^2/(Z_2 \times Z'_2)$  model, which has fixed lines with reduced gauge symmetry. Although  $b = 0$  is guaranteed for this model due to the 6D  $N = 2$  supersymmetry, the  $b'_i$  do not vanish, and moreover are not universal. As a consequence the gauge couplings experience power-law (linear) differential running above the compactification scale. Threshold corrections to  $\sin^2 \theta_w$  become quite large if the cutoff is taken to be much larger than the compactification scale, and we estimate this correction to be  $\sim (2 - 3)\%$  for  $M_*R \sim 3$ . In the  $T^2/(Z_2 \times Z'_2)$  model, consistency with low-energy data most likely requires some degree of cancellation between threshold corrections coming from unknown cutoff-scale physics and this correction arising from KK modes.

## 7 Conclusions

In this paper we constructed three supersymmetric  $SO(10)$  theories in which the gauge symmetry is broken by orbifold compactification. Unlike in the  $SU(5)$  case, where a single extra dimension is sufficient for breaking the gauge symmetry, we find in the  $SO(10)$  case that at least two extra dimensions are required to break the symmetry to  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ .

Since we are led to consider 6D theories, important obstacles absent in the 5D case arise. One is the potential for bulk anomalies, present only in theories with even spacetime dimensions. A second is the possibility of subspaces with reduced gauge symmetry that spoil successful gauge coupling unification. In 5D theories the subspaces with reduced gauge symmetry are 4D, giving only a logarithmic threshold correction to  $\sin^2 \theta_w$ . Depending on the orbifold, 6D theories may have 5D subspaces on which the gauge symmetry is broken, leading to a power-law correction to  $\sin^2 \theta_w$ . A different challenge, particular to  $SO(10)$  theories, is that the orbifold compactification generically does not break the gauge symmetry all the way down to the standard model gauge group, as it can in 5D  $SU(5)$  theories:  $U(1)_X$  is left unbroken, and the right-handed neutrinos are massless.

The first model is constructed on a  $T^2/Z_2$  orbifold, and possesses 6D  $N = 1$  supersymmetry. The structure of the orbifold is such that the full  $SO(10)$  is realized everywhere but on 4D fixed points, guaranteeing that threshold corrections to  $\sin^2 \theta_w$  are under control. The irreducible bulk gauge anomalies can be canceled by adding two bulk hypermultiplets in the fundamental representation (containing the two Higgs doublets of the MSSM), with the option of adding additional pairs of hypermultiplets, each pair containing one spinor and one fundamental. This allows one to build models in which doublet-triplet splitting is naturally realized, and in which  $U(1)_X$  is broken by the VEV of an  $SU(5)$  singlet localized on an  $SU(5) \otimes U(1)_X$  preserving brane, giving rise to masses for the right-handed neutrinos either through direct interaction (for the case of matter in the bulk), or by integrating out bulk states (for the case of matter on a fixed point). Even for the case of matter localized to an  $SO(10)$  preserving fixed point, unwanted GUT fermion mass relations can be corrected through mixing with bulk states.

Unfortunately, for this  $N = 1$  model mixed gauge-gravitational anomalies do not cancel. This motivates us to consider theories with 6D  $N = 2$  supersymmetry, for which anomaly cancellation is automatic. The  $T^2/Z_2$  orbifold cannot be used to build such a theory because the fixed points have too much supersymmetry left over after orbifolding: 4D  $N = 2$  rather than 4D  $N = 1$ . Instead we used a  $T^2/Z_6$  orbifold that gives two points with 4D  $N = 1$  supersymmetry, and  $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$  and  $SU(5) \otimes U(1)_X$  gauge symmetries, respectively. As in the  $T^2/Z_2$  model, the  $SO(10)$  is broken only at 4D points and so there is no power-law differential running of the gauge couplings above the compactification scale. To avoid light colored Higgs states the most convenient choice is to put matter and Higgs on the 4-2-2 point. The  $U(1)_X$  symmetry can be broken by the VEV of an  $SU(5)$  singlet on the 5-1 brane, but there is no clear way of communicating this breaking to the 4-2-2 brane to give right-handed neutrino masses; also, there is no clear way of relaxing  $SO(10)$  fermion mass relations. The alternative of breaking  $U(1)_X$  on the 4-2-2 fixed point is also problematic as it leads to a vacuum alignment problem and massless Goldstone states: canceling anomalies by restricting the bulk matter content to be 6D  $N = 2$  supersymmetric makes  $U(1)_X$  communication and attainment of realistic fermion

masses a challenge because it makes the bulk less accessible.

We explored a resolution to this problem in the third model, on  $T^2/(Z_2 \times Z'_2)$ . Although the bulk is again taken to possess 6D  $N = 2$  supersymmetry, this orbifold has 5D lines, fixed under one  $Z_2$  but not the other, which possess only 6D  $N = 1$  supersymmetry. These lines connect 4D points with 4D  $N = 1$  supersymmetry, where interactions can arise. Matter multiplets may be introduced on these 5D lines without spoiling bulk anomaly cancellation, so this set up yields a number of possibilities for realistic models. In particular, natural doublet-triplet splitting is accommodated by appropriate placement of Higgs multiplets on these lines, communication of  $U(1)_X$  breaking is now straightforward, and realistic fermion masses can be attained.

The trade-off is that this model has 5D surfaces that do not preserve  $SU(5)$ , leading to linear running of the gauge couplings relative to one another above the compactification scale: the fixed lines are welcome for certain model building purposes but damaging for the prediction of  $\sin^2 \theta_w$ , especially if the cutoff is taken much larger than the compactification scale. An attractive  $N = 2$  model would be one with only  $SO(10)$  or  $SU(5) \otimes U(1)_X$  preserving fixed lines. The challenge is to realize this situation while breaking the gauge symmetry down to the standard model group, and while accommodating natural doublet-triplet splitting, right-handed neutrino mass generation, and realistic fermion mass matrices. It will be interesting to pursue a fully realistic model along these lines in the future.

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